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LEFT PATTERN MATCHING PREDICTOR IS OPTIMAL OVER BERNOULLI SOURCE MODELS

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Abstract: We show that the Left Pattern Matching predictor is perfect and optimal over Bernoulli source models. We prove the proposition by induction on the size of the text. We reduce the size of a text by using the simple trick of enlarging the size of the alphabet *via* the use of bucket symbols. We prove the induction on an enlarged problem on weighted pattern matching with the use of pairing cost matrix.

Key-words: Information Theory, Predictor, Pattern Matching, Bernoulli sources, Alphabet

(Résumé : tsvp)

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Le prédicteur à recherche du meilleur motif à gauche est optimal sur les sources Bernoulli

Résumé : Nous montrons que le prédicteur à recherche de motif à gauche est parfait et optimal sur les sources Bernoulli. Nous prouvons cette propriété par récurence sur la taille du texte soumis. Nous réduisons la taille du texte par une expansion judicieuse de l'alphabet, grace à l'usage des symboles réunis en paquet. Nous prouvons l'optimalité du prédicteur dans un cas général où la recherche du meilleur motif est pondéré par une matrice de coût.

Mots-clé : Théorie de l'Information, Prédicteur, Recherche de motif, Sources Bernoulli, Alphabet

1 Introduction

We consider an infinite random sequence $x_1x_2\dots x_n\dots$ generated according to an unknown random source process based on a finite alphabet. We denote X_n the prefix of length n of this sequence. A predictor is a function $P(X_n)$ of X_n which strives to predict x_{n+1} knowing only X_n. A perfect predictor should be $P(X_n)=x_{n+1}$. Perfect predictors are only possible on randomless sources. Globally perfect predictors are those which satisfies

$$\Pr\{P(X_n)=a\}=\Pr\{x_{n+1}=a\}$$

or asymptotically globally perfect predictors where:

$$\lim_{n\to\infty}|\Pr\{P(X_n)=a\}-\Pr\{x_{n+1}=a\}|=0$$

This should not be confused with optimal predictors which minimizes $\Pr\{P(X_n)\neq x_{n+1}\}$ or with asymptotically optimal predictor which tends to minimize this probability when $n\to\infty$. Remark that with Bernoulli source models, globally perfect predictors are also optimal and *vice versa*. It is true that for all kinds of sources optimality implies global perfection but the contrary is not always true. For example a predictor can be globally perfect for Markov sources but not optimal, since the probability of transition from the last symbol may not be respected.

An example of simple predictor is the one which repeats the last character of X_n: $P(X_n)=x_n$. Obviously this predictor is optimal and global perfect for Bernoulli sources. The predictor is asymptotically globally perfect for Markov sources but not asymptotically optimal since the transition to the last character is not considered, in particular for Markov sources where $\Pr\{x_{n+1}=x_n\}=0$.

Before asymptotically optimal predictors are the asymptotically locally optimal predictors: for all integer k: for all subwords σ_k of length k generated by the random source:

$$\lim_{n\to\infty}|\Pr\{P(X_n)=a\mid x_{n-k}\dots x_n=\sigma_k\}-\Pr\{x_{n+1}=a\mid x_{n-k}\dots x_n=\sigma_k\}|=0$$

It is not difficult to prove that asymptotically locally optimal predictors are asymptotically optimal for sources satisfying mixing conditions.

The applications are numerous and may not need to be listed in details. There are application in control, forecast, financial data, communication, *etc.*

In the present note we investigate Pattern Matching based predictor [1, 2]. A pattern matching predictor scans the sequence X_n in order to find the largest suffix which has an exact copy inside X_n. Let k be the position in X_n of the last character of this copy, then the prediction is therefore the value the character just after: $P(X_n)=x_{k+1}$.

In the case where the largest suffix has more than one clone inside X_n there is an ambiguity. If one select the first copy then we are in left largest pattern matching version (LPM predictor), if we select the last copy we are in right largest copy (RPM predictor).

In the extremal case where no largest suffix can be found: *i.e* when the last character of X_n has a value which appears nowhere else in the sequence, therefore by convention $P(X_n)=x_2$ for LPM and $P(X_n)=x_n$ for RPM.

It will be proven in a next paper with Wojciech Szpankowski that Pattern Matching predictors are all asymptotically locally optimal for a very large class of random sources (mixing model). The aim of this short note is to prove that LPM predictor is globally perfect (and therefore optimal) for Bernoulli model, and RPM is not.

2 Finite examples

We takes the last paragraph of the previous section and we apply to it left pattern matching predictor. Correctly predicted substrings are underlined. The three first letters of the text were provided as initialization input in the predictor algorithm.

IT WILL BE PROVEN IN_A NEXT PAPER THAT_PATTERN_MATCHING PREDICTORS
ARE ALL_ASYMPTOTICALLY LOCALLY OPTIMAL FOR A_VERY LARGE_CLASS_OF
RANDOM SOURCES_(MIXING_MODEL) THE_AIM OF_THIS_SHORT_NOTE IS_TO PROVE
THAT LEFT_PATTERN MATCHING PREDICTOR IS_GLOBALLY_PERFECT (AND THEREFORE
OPTIMAL)_FOR BERNOULLI MODEL AND RIGHT_PATTERN MATCHING PREDICTOR
IS NOT

Contrary to apparence, the matching between the predicted text and the original text is very good: more than one third of the letters (exactly 0.3391) were correctly predicted, which is much better than a random guess over the 25 different symbols used above.

Let below a random text obtained by a Bernoulli source symbol using the same symbol proportion as in the previous text. The correct prediction probability drops down to 0.064.

RNXOROIRTAOIARORLTR HG R N RRA LPTPOE PTDTAATOMYFS SLCRNS_RARA_COHLT
FRT RHAGTTBI O N_EECUM T L YA EAPORBE_ TTIUMFPXEBM NMEIEP IPYLTAUILNTT
FIGMDPNPNNTDR TF REAO LLEREMGPIORIT IE ETAREWI NOL AALTNATIATR_
IAIOMSEEPFA AF MO I TOSOLNTMMRAIEOA IEA IRIFLEMA(ELIID A TRRLAHUO-
HYO TY TOPR LNOINFPIPST ALHOTYSADAASIEMATEEMGANPPI AXNMTRTROLASAOTMLFDT

Let us consider a binary alphabet $\{a, b\}$. Below we display the LPM and RPM prediction over a binary alphabet and all strings of length 2 and 3.

string	LPM prediction	RPM prediction
aa	a	a
ab	b	b
ba	a	a
bb	b	b
aaa	a	a
aab	a	b
aba	b	b
baa	a	a
bba	b	a
bab	a	a
abb	b	b
bbb	b	b
$aaaa$	a	a
$aaab$	a	b
$aaba$	a	b
$abaa$	b	a
$baaa$	a	a
$aabb$	b	b
$abab$	a	a
$abba$	b	b
$baab$	a	a
$baba$	b	b
$bbaa$	a	a
$abbb$	b	b
$babb$	a	b
$bbab$	b	a
$bbba$	b	a
$bbbb$	b	b

Let $L_n(p)$ be the probability of predicting a over a Bernoulli text of length n where a occurs with probablity p and b with probability $1-p$. $L_n(p)$ is a polynomial of p with degree smaller than or equal to n. We compute $L_n(p)$ for $n = 2, 3, \ldots$,

$$\begin{array}{rcl} L_2(p) & = & p \\ L_3(p) & = & p \\ L_4(p) & = & p \\ L_5(p) & = & p \\ L_6(p) & = & p \\ L_7(p) & = & p \\ L_8(p) & = & p \\ L_9(p) & = & p \\ L_{10}(p) & = & p \end{array} \quad (1)$$

Therefore LPM is optimal for those values of n. The object of this note is to prove that this result is true for all n in a general way. But before we will prove that RPM predictor is *not* optimal.

Let $R_n(p)$ be the probability that the prediction is equal to symbol a over a Bernoulli text of length n where a occurs with probability p and b with probability $1-p$. $R_n(p)$ is a polynomial of p with degree smaller than or equal to n. Easy computations yield:

$$\begin{array}{rcl} R_2(p) & = & p \\ R_3(p) & = & 2p-3p^2+2p^3 \\ R_4(p) & = & 2p-3p^2+2p^3 \\ R_5(p) & = & 3p-9p^2+16p^3-15p^4+6p^5 \\ R_6(p) & = & 3p-10p^2+20p^3-20p^4+8p^5 \\ R_7(p) & = & 4p-18p^2+46p^3-65p^4+54p^5-28p^6+8p^7 \\ R_8(p) & = & 4p-20p^2+59p^3-100p^4+103p^5-63p^6+18p^7 \\ R_9(p) & = & 5p-31p^2+109p^3-225p^4+293p^5-245p^6+130p^7-45p^8+10p^9 \\ R_{10}(p) & = & 5p-33p^2+125p^3-282p^4+411p^5-399p^6+258p^7-108p^8+24p^9 \end{array} \tag{2}$$

Since $R_n(p) \neq p$ for $n = 3, 4, \ldots$ it is clear that RPM is not optimal (or globally perfect) for these values of n. It will be proven in a next paper that this property is true only *asymptotically.*

Theorem 1 *RPM predictor is not optimal for Bernoulli models when $n > 2$.*

Proof: We consider that string X_n is from a Bernoulli source where the probability p of symbol a occurence is very small. Our aim is to make the Taylor expansion of $R_n(p)$ with respect to p up to first order.

When generating string X_n, we have the following event breakdown:

1. With probability $1 - np + O(p^2)$, X_n contains only b;
2. with probability $np + O(p^2)$, X_n contains only one a with the n position equiprobable;
3. other configurations occur with probability $O(p^2)$.

In case 1, the LPM and RPM prediction is b. In case 2 the RPM prediction is a as long the longest continuous sequence of b's in X_n is the one just before the position of symbol a, namely when a position varies between $\lfloor \frac{n}{2} \rfloor$ and n. Therefore

$$R_n(p) = \lceil \frac{n}{2} \rceil p + O(p^2)$$

which confirms the above numerical computations. Therefore RPM predictor is not perfect (optimal) for Bernoulli models.■

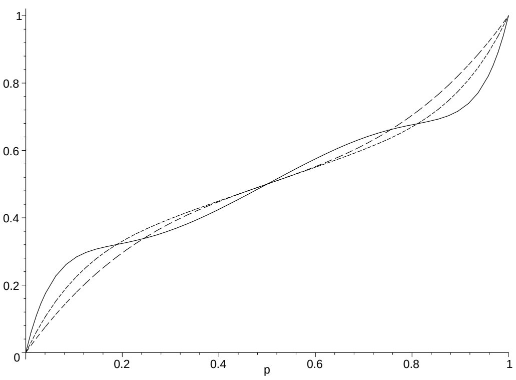

Figure 1: Plot display of $R_3(p)$, $R_6(p)$, $R_{12}(p)$

Remark: In case 2 LPM model predicts a only when the position of symbol a is exactly $\lfloor \frac{n}{2} \rfloor$. Therefore $L_n(p) = p + O(p^2)$ for all n, which is a good hint for the proposition that $L_n(p) = p$.

The asymptotical optimality of Right Pattern Matching predictor is equivalent to the fact that for any fixed $p \in [0,1]$:

$$\lim_{n\to\infty} R_n(p) = p \ . \tag{3}$$

But interestingly enough this convergence does not imply that the coefficients of polynomials $R_n(z)$ converge to the coefficients of the polynomial z (or polynomial identity). In other words we only have the convergence of function $R_n(p)$ to p in sense of measure on $[0,1]$, and the convergence does not hold outside this interval. In figure 1 we display the plot figure of functions $R_3(p)$, $R_6(p)$ and $R_{12}(p)$. The convergence of $R_n(p)$ to p is sensible for p around 1/2 but

3 Use of pairing matrix

3.1 The permutation identities

The proof will proceed by recursion on n. Given an alphabet of size V: $\{a_1, \ldots, a_V\}$, we denote $X_{n_1,\ldots,n_V}$ a random sequence which respectively contains n_1 symbols a_1, n_2 symbol

a_2, ..., n_V symbols a_V, all permutations being equiprobable. We are going to prove that for every symbol a_i in the alphabet:

$$\Pr\{P(X_{n_1,\ldots,n_V}) = a_i\} = \frac{n_i}{n} \tag{4}$$

with $n = n_1 + \cdots n_V$. We call the above identities, the permutation identities. Just by equivalence with polynomial identities it is clear that permutation identities, when true for all combinations within a given n, are strictly equivalent to global optimality for all Bernoulli sources for sequence of length n.

3.2 Weighted Pattern Matching

The strategy of the proof is based on induction on n. But before proceeding we will need to enlarge the problem and we introduce the weighted pattern matching predictor (WPM predictor). We introduce a pairing matrix $\mathbf{C} = [c_{ij}]$ which satisfies the following properties:

- all coefficients c_{ij} are positive;
- the matrix is symmetric: $c_{ij} = c_{ji}$;
- for all i, j: $c_{ij} = \max_k\{\min\{c_{ik}, c_{kj}\}\}$

The last property can be called the min-max injectivity: we have $\mathbf{C} \star \mathbf{C} = \mathbf{C}$ in the algebra where operator "max" plays the role of addition and operator "min" plays the role of multiplication. Notice that operator “min” distributes over operator “max” (and vice versa), therefore (“max”, “max”) forms an algebra.

Matrix identity plays the role of pairing matrix in the basic pattern matching predictor problem.

Remark: The matrix $\mathbf{C}$ is diagonally dominated, *i.e.* $c_{ii} \geq c_{ij}$ for all i, j. The coefficient c_{ii} is the weight of symbol a_i.

The pairing weight of two words X and Y is the sum of the symbol weights of each characters in the largest common suffix to X and Y plus the coefficient c_{ij} provided that the character before the prefix in X is a_i and the character before the prefix in Y is a_j. If no character is present before the prefix in X or in Y, by convention we add nothing.

The weighted pattern matching consists into finding the copy of the largest suffix which maximizes the pairing weight. In case of ambiguity the first largest weighted copy is taken. And the WPM predictor takes the values of the first character after the largest weighted copy.

3.3 Single and paired symbols

Definition 1 *Symbol a_i is single in string X_n when a_i occurs only once in X_n.*

Definition 2 *Symbol a_i is paired to symbol a_j in string X_n when:*

1. *a_i and a_j are both single in X_n;*
2. *a_j is the only symbol different of a_i which maximizes c_{ik} over all symbols a_k present in X_n.*

Lemma 1 *Let be X_n be an arbitrary string. If symbol a_i is paired to symbol a_j in X_n, then the reverse is true: symbol a_j is paired to symbol a_i in X_n.*

Proof: Assume a_i is paired to a_j in X_n. Let $\mathcal{S}(X_n)$ be the set of the indexes of the symbols present in X_n. Therefore $\{i,j\} \subset \mathcal{S}(X_n)$. For all $k \in \mathcal{S}(X_n)$, k different of i and j we have $c_{ik} < c_{ij}$. Suppose now that a_j is *not* paired to a_i. Therefore there exists $k \in \mathcal{S}(X_n)$ different of i and j such that $c_{jk} \geq c_{ij}$. Thus the triangular inequality of the pairing matrix would lead to $c_{ik} \geq \min\{c_{ij}, c_{kj}\} = c_{ij}$ which contradicts the fact that $c_{ij} > c_{ik}$. ■

3.4 Splitting and factoring

The proposition that we will prove is that whatever the pairing matrix:

$$\Pr\{P(X_{n_1,\ldots,n_V}) = a_i\} = \frac{n_i}{n} \tag{5}$$

Equivalently the probability of predicting symbol a_i from sequence $X_{n_1,\ldots,n_V}$ is the same as the probability of having the first character of $X_{n_1,\ldots,n_V}$ equal to a_i. This remark will be of some utility later.

The proposition is obvious for $n=2$, $n=3$ is left to the reader. We will prove the proposition by induction. To this end we will split (5) in the following identity:

$$\Pr\{P(X_{n_1,\ldots,n_V}) = a_i\} = \sum_j \frac{n_j}{n} \Pr\{X_{n_1,\ldots,n_j-1,\ldots,n_V} a_j = a_i\} \tag{6}$$

Therefore we will study the distribution of $P(X_{n_1,\ldots,n_V} a_i)$. To simplify we identify $i=1$. The case where symbol a_1 is single ($n_1 = 0$) in $X_{n_1,\ldots,n_V} a_1$ can be handled directly without induction. Indeed the work consists into finding the next symbols to the leftmost symbol in $X_{n_1,\ldots,n_V}$ which minimize c_{1j}.

Lemma 2 *If a_1 is single but not paired in $X_{n_1,\ldots,n_V} a_1$, then for all i:*

$$\Pr\{X_{n_1,\ldots,n_V} a_1 = a_i\} = \frac{n_i}{n} \tag{7}$$

with $n = n_1 + \cdots n_V$.

If a_1 is paired to a_j in $X_{n_1,\ldots,n_V} a_1$, then (7) is true for all i distinct of 1 and j. And

$$\Pr\{X_{n_1,\ldots,n_V} a_1 = a_i\} = 0$$

but in counterpart

$$\Pr\{X_{n_1,\ldots,n_V} a_1 = a_1\} = \frac{1}{n}$$

Proof: the proof is straightforward and left to the reader. ∎

For all case where $n_1 > 0$ we will prove by induction for all i:

$$\Pr\{X_{n_1,\dots,n_V} a_1 = a_i\} = \frac{n_i}{n}$$

where $n = n_1 + \cdots + n_V$. Or, equivalently, that $P(X_{n_1,\dots,n_V} a_1)$ has same distribution as the first character in $X_{n_1,\dots,n_V} a_1$.

Corollary 1 *If the induction hypothesis is true for n, then (5) is true for n.*

Proof: Let a_k be an impaired symbol in $X_{n_1,\dots,n_V}$. Since (7) is assumed to be true in all case we have:

$$\begin{aligned} \Pr\{P(X_{n_1,\dots,n_V}) = a_k\} &= \textstyle\sum_j \frac{n_j}{n} \Pr\{X_{n_1,\dots,n_j-1,\dots,n_V} a_j = a_k\} \\ &= \textstyle\sum_{j\neq k} \frac{n_j}{n}\frac{n_k}{n-1} + \frac{n_k}{n}\frac{n_k-1}{n-1} = \frac{n_k}{n} \end{aligned} \tag{8}$$

Let a_i be a paired symbol in $X_{n_1\dots,n_V}$. Let a_j the symbol paired to a_i. An obvious consequence is that $n_i = n_j = 1$ and by lemma 2

$$\begin{aligned} \Pr\{X_{n_1,\dots,n_i-1,\dots,n_V} a_i = a_i\} &= \tfrac{1}{n-1} \\ \Pr\{X_{n_1,\dots,n_j-1,\dots,n_V} a_j = a_i\} &= 0 \end{aligned}$$

For all $k \neq i, j$ we have

$$\Pr\{X_{n_1,\dots,n_k-1,\dots,n_V} a_k = a_i\} = \frac{1}{n-1}$$

In summary:

$$\begin{aligned} \Pr\{P(X_{n_1,\dots,n_V}) = a_k\} &= \textstyle\sum_j \frac{n_j}{n} \Pr\{X_{n_1,\dots,n_j-1,\dots,n_V} a_j = a_k\} \\ &= \textstyle\frac{1}{n}\frac{1}{n-1} + \sum_{k\neq i,j} \frac{n_k}{n}\frac{1}{n-1} = \frac{1}{n} \end{aligned} \tag{9}$$

∎

In conclusion it suffices to prove by induction identity (7) for all vectors $(n_1, \dots, n_V)$ such that $n_1 > 0$ to obtain identity (5) valid for all $(n_1, \dots, n_V)$.

4 Bucket expansion of the alphabet

We consider the sequence $Y_{n+1} = X_{n_1,\dots,n_V} a_1$ with $n_1 > 0$. The induction argument consists into decrementing the number $n + 1$ of characters in Y_n to make it smaller or equal to n. To this end we naturally needs to expand the size of the alphabet. We do it by defining new symbol as group of old symbols. A bucket symbol is of the form σa_1 where σ is a finite sequence σ of characters all different of a_1. We restrict to sequences of size smaller than n. The smallest bucket symbol is a_1 when σ is empty.

Below is an example with binary alphabet $\{a, b\}$. We consider strings ending with symbol a. We display in the table below the list of bucket symbols:

bucket symbol	symbol name
a	β_0
ba	β_1
bba	β_2
$bbba$	β_3
$bbbba$	β_4
$\ldots$	

Let $X_n = abababbabaaba$ be a string. The bucket factorization of string X_n is as follows

$$X_n = (a)(ba)(ba)(bba)(ba)(ba) \ .$$

Therefore the equivalent $\widetilde{X}_n$ of X_n written in bucket symbols is

$$\widetilde{X}_n = \beta_0\beta_1\beta_1\beta_2\beta_1\beta_1 \ .$$

The pairing matrix $\widetilde{\mathbf{C}}$ is defined as follow. Let $\sigma_1 a_1$ and $\sigma_2 a_1$ be two bucket symbols. The coefficient of $\widetilde{\mathbf{C}}$ corresponding to the pair $(\sigma_1 a_1, \sigma_2 a_1)$ is equal to the pairing weight of the two words $X = a_1\sigma_1$ and $Y = a_1\sigma_2$. Notice that in both X and Y, symbol a_1 is put on the first place, in order to include border effect. Checking min-max injectivity is left to the reader.

Also left to the reader the verification that the weighted pattern matching on Y_n transcripted in bucket symbols finds the same positions for the copies of the largest weighted suffix with the new pairing matrix $\widetilde{\mathbf{C}}$ as on Y_n writen with its original symbols with the old matrix $\mathbf{C}$. Notice that the weights are just the same but shifted by the constant term c_{11}. Therefore if $BP(Y_n)$ is the bucket prediction of Y_n, then the selected bucket character starts on the same position as the selected symbol with the old prediction. Therefore if $(BP(Y_n))_1$ is the first symbol of the bucket prediction we have $(BP(Y_n))_1 = P(Y_n)$.

Now remains to factorize Y_n in bucket symbols. It is not difficult to see that Y_n will contain exactly $n_1 + 1$ bucket symbols. Let W the size of the bucket alphabet and let denote the bucket symbols by $b_1, \ldots b_W$, and let $\widetilde{X}_{m_1,\ldots,m_W}$ be a random permutation of m_1 symbols b_1, m_2 symbols b_2, …, m_W symbols b_W. It need very little effort to see that the distribution over Y_n factorize into several classes of the form $\widetilde{X}_{m_1,\ldots,m_W}$.

Since the number of bucket symbols in $\widetilde{X}_{m_1,\ldots,m_W}$ is exactly $n_1 + 1$ and $n_1 + 1 \leq n$ (excepted in trivial cases easy to handle), we can apply the induction hypothesis. Therefore (5) holds on each class, therefore the distribution of $BP(\widetilde{X}_{m_1,\ldots,m_W})$ is the same as the distribution of the first bucket character in $\widetilde{X}_{m_1,\ldots,m_W}$. Therefore it comes that the distribution of $BP(Y_n)$ is the same as the distribution of the value of the first bucket character in Y_n. Since the first character of the first bucket symbol of Y_n is the first character of Y_n we obtain (7) by induction, and finally (5).

Therefore LPM predictor is optimal for Bernoulli sources.

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